Spectral flatness factor and 'intermittency' in turbulence and in non-linear noise

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The flatness factor F of the signal transmitted through a band-pass filter has been measured for the turbulence in a free shear layer and for a squared Gaussian noise. They both show flatness factor increasing with centre frequency f_c. In the turbulence, band-passed signals look intermittent and have larger F than the full signal, but in the squared noise, band-passed signals all have smaller F than the full signal although they look more intermittent.

It is shown analytically that the derivative of a smoothed, squared Gaussian noise may have flatness factor either greater or less than the undifferentiated signal.

Batchelor & Townsend (1949) and Sandborn (1959) have shown experimentally that the fine structure of turbulence in grid flow, wakes and boundary layers tends to be intermittent (see also Batchelor 1949). The former experiments used essentially high-pass filters (the effect of successive differentiations). The latter used band-pass filters. A possible measure of intermittency in an intermittent variable b is the amount by which the flatness factor

$$F \equiv \overline{b^4}/(\overline{b^2})^2 \tag{1}$$

(also called kurtosis) exceeds the value 3·0 that is appropriate to a variable with Gaussian (or 'normal') probability density. Of course, this measure is useful only if the 'on' part of the intermittent variable is roughly Gaussian.

For the turbulence, qualitative explanation in terms of strong localization of vorticity into sheets and/or lines has been offered convincingly by Batchelor & Townsend. This vortex stretching is a result of the non-linear, convective acceleration term in the Navier–Stokes equations, so it seemed relevant to see whether other (simpler) non-linear random systems show similar spectral kurtosis properties. No analogy is implied between the squared Gaussian noise and turbulence.

Turbulent shear layer

A constant current hot-wire anemometer circuit was used. The hot-wire sensing element was mounted 12 in. downstream of the exit of an 18 in. square duct, on the extrapolation line of the wall (figure 1). The inside duct boundary layer was turbulent and about $\frac{1}{2}$ in. thick. The free-stream velocity was 25 ft./sec; the mean velocity at the wire was about half of that. No general intermittency at the wire was detectable on an oscilloscope. The flatness factor of the full turbulence signal (component along the flow) was 3·0.

Figure 2 is a block diagram of the measuring devices. The variable band-pass filter had a half-power point band width equal to half the centre frequency: $\Delta f/f_c \approx 0{\cdot}5$. Sandborn shows typical oscillograms of the full turbulence signal and a filtered turbulence signal which shows some intermittency.

The flatness factors measured for a range of f_c is included in figure 4. These results are in qualitative agreement with those of Sandborn; the band-passed signal has flatness factor above 3·0 for all f_c, and increases toward larger f_c.

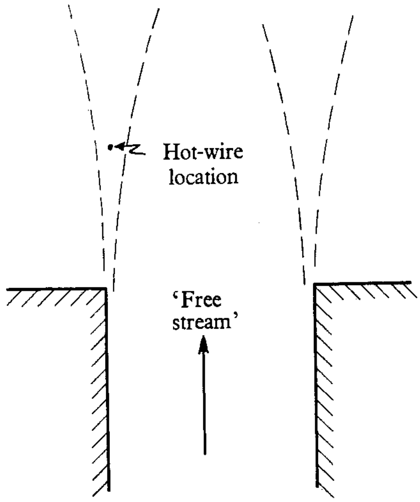

FIGURE 1. Experimental arrangement.

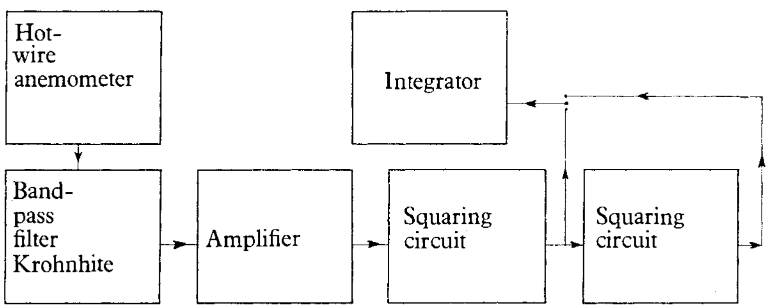

FIGURE 2. Block diagram of the measuring devices.

Gaussian noise

An electronic noise generator (H. H. Scott, type 811-A, ASA Band) was used as source of random signal with Gaussian probability density function. This was put in place of the hot-wire and its amplifier in the circuit of figure 2. As indicated in figure 4, this had a band-passed flatness factor close to 3·0. Neither of the corresponding oscillograms in figure 3, plate 1, looks particularly intermittent.

Squared Gaussian noise

As a typical simple non-linear process we chose an instantaneous squaring device. With Gaussian input, the output is a non-Gaussian random signal. The output of the noise generator was passed through an approximate squaring circuit and the d.c. component was removed with a high-pass filter (figure 5), cutting off below 10 c/s.

The total signal does not look especially intermittent (figure 3), although its flatness factor is 9·1.* In contrast, the band-passed signals do have a more intermittent appearance (figure 3), but coupled with appreciably smaller flatness factors (figure 4).

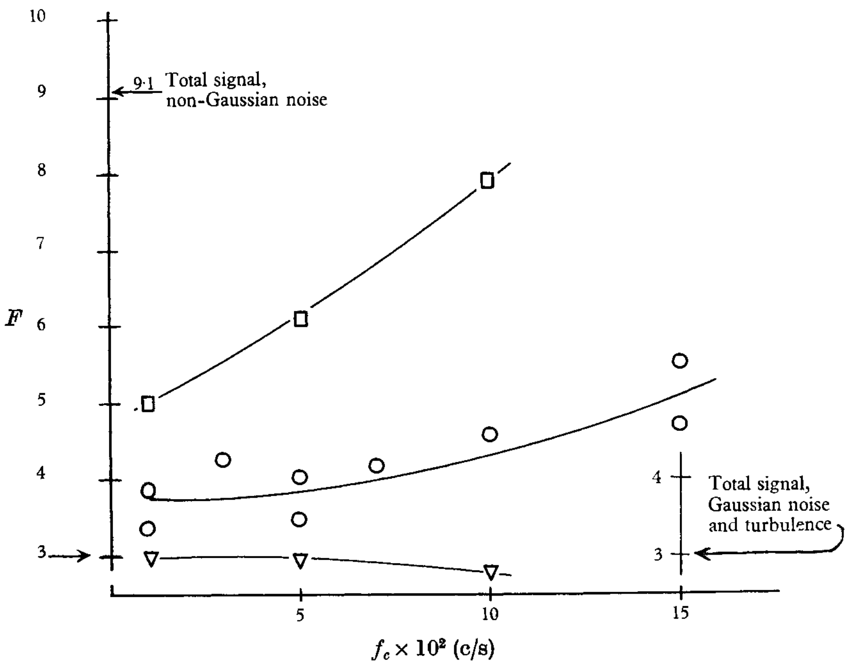

FIGURE 4. Flatness factors. ○, Free jet turbulence; □, non-Gaussian noise; ▽, Gaussian noise; $\Delta f/f_c = 0{\cdot}5$ for all cases.

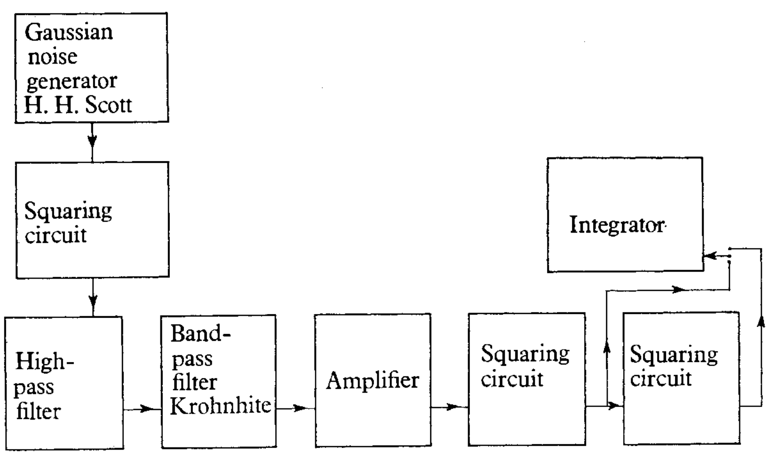

FIGURE 5. Block diagram showing use of high-pass filter.

* The theoretical value for a squared Gaussian with d.c. removed is 15. The value 9 corresponds theoretically to the power 1·7.

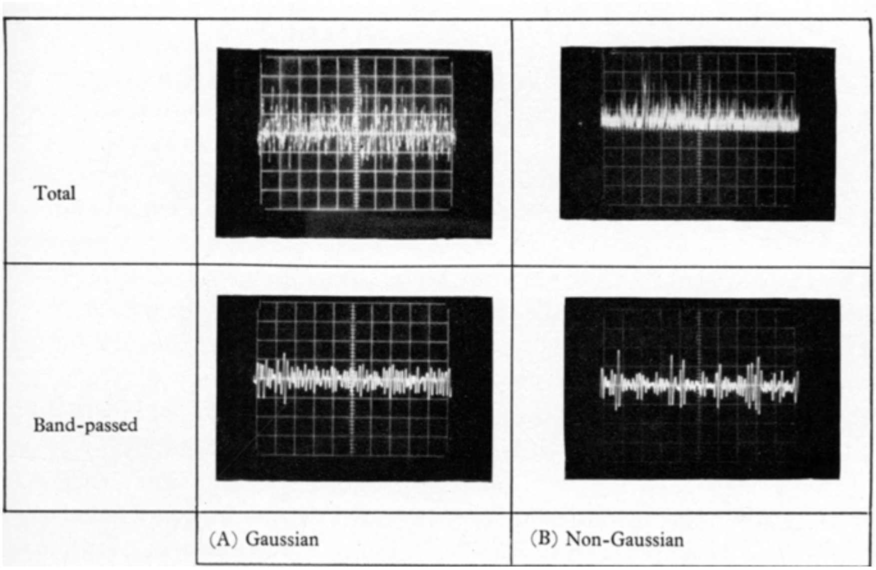

FIGURE 3. Oscillograms of 'noise'.

These results differ from the turbulence case in the two following ways.

(1) The band-passed signals here have smaller F than the total signal; for turbulence this is reversed.

(2) Here the signal with larger flatness factor (i.e. the total signal) is less intermittent in appearance; for turbulence this is reversed.

A point of similarity with the turbulence signal is that F increases with increasing f_c in both.

It seems desirable to pursue this sort of measurement through other classes of non-linear operations, especially non-linear differential equations with random forcing functions. It may turn out that the response will have flatness factor near 3 (as in some turbulent velocities) simply because it is in a sense a sum. This brings into play the effects that give us Central Limit Theorems in probability theory. In such a case we might then expect departure from Gaussian properties in the derivatives of the solution, hence in the higher frequencies. There seems to be as yet no corresponding qualitative argument to rationalize the fact that narrow-band signals in the lower frequency range (of turbulence) also have flatness factor larger than the full signal.

Finally, all such measurements should be made with a range of relative band widths extending to much smaller values than used so far.

A related exercise

Suppose $\theta(x)$ is a stationary 'Gaussian process' in the sense that the values chosen at n different points ($1 \leqslant n = \infty$) are jointly Gaussian. As we have seen, $[\theta^2(x) - \overline{\theta^2}]$ is a non-Gaussian random variable.

There is both experimental (Iribe 1949) and theoretical (Kac & Siegert 1947) evidence that the averaging ('smoothing') of a non-Gaussian variable tends to make it more nearly Gaussian. As indicated above, this is doubtless related to the conditions for Central Limit Theorems to be applicable.

Define the smoothed non-Gaussian variable

$$R_\xi(x) \equiv \frac{1}{\xi}\int_x^{x+\xi} [\theta^2(x_1) - \overline{\theta^2}]\, dx_1. \tag{2}$$

For $\xi \to 0$, it is obvious that

$$\lim_{\xi\to 0} [R_\xi(x)] = \theta^2(x) - \overline{\theta^2}. \tag{3}$$

For $\xi \to \infty$, we expect $R_\xi(x)$ to approach a Gaussian process (Kac & Siegert 1947).

Focusing attention on the flatness factor, $F_R \equiv \overline{R_\xi^4}/(\overline{R_\xi^2})^2$, we see that

$$\lim_{\xi\to 0} F_R = 15, \tag{4}$$

whereas we expect

$$\lim_{\xi\to \infty} F_R = 3. \tag{5}$$

The statistical properties of $R_\xi(x)$ are functionals of the properties of $\theta^2(x)$, so general results are difficult to obtain. However, one particular function of $R_\xi(x)$ is not only relevant here but also accessible:

$$R'_\xi(x) \equiv \frac{d}{dx} R_\xi(x) = \frac{1}{\xi}[\theta^2(x+\xi) - \theta^2(x)]. \tag{6}$$

With the condition that $\theta(x)$ is Gaussian, computation yields

$$F_{R'} \equiv \frac{\overline{R_\xi'^4}}{(\overline{R_\xi'^2})^2} = 9, \tag{7}$$

independent of ξ. This shows that for small enough ξ, the differentiated variable $R'_\xi(x)$ has smaller F than the undifferentiated one $R_\xi(x)$, whereas for large ξ it has larger F. The former situation corresponds to the measurements on squared Gaussian noise with no smoothing ($\xi = 0$). The latter result is qualitatively like that observed for turbulent velocity. We recall that differentiation is like a high-pass filter.

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